Tradable performance standards in a dynamic context

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ABSTRACT
Many sectors of the economy that are targets of emissions reduction policy tend to be subject to price-responsive demand, long-lived capital, capacity constraints, and foresighted decision-making. I explore these features together, in conjunction with a tradable performance standard (TPS). First, I provide a complete characterization of the short-run and steady-state output responses analytically. Second, I validate the intermediate analytical results, explore the dynamics of the transition from pre- to post-policy steady-state, and discuss the welfare implications using a stylized numerical equilibrium model calibrated to a representative electricity sector. I show that the difference in the present value of total social surplus gains between a TPS and a period-over-period damage equivalent cap (CAP) is small relative to total social surplus gains from either policy. Most interestingly, under all but the smallest discount rates, the value of the steady-state perpetuity under the TPS is greater than the CAP.

JEL classifications: C61, H23, Q41, Q48, Q52, Q58
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1 Introduction

Rate-based standards come in many forms and are characterized by requiring the sum of emissions from all sources divided by total output to be less than some pre-specified intensity target. With a tradable performance standard (TPS), the average emissions rate is fixed. For production with emissions rates below the standard, operation creates permits which can be sold. For production with emissions rates above the standard, operation requires the purchase of permits to cover the difference. The equilibrium that arrives as a result of trading these permits determines the price of emissions and total amount of emissions, such that the average emissions intensity for the entire industry equals the TPS policy target (Fischer, 2001). A consequence of the TPS is that while the pre-specified intensity standard acts as an implicit tax on dirty or more emissions-intensive inputs to production of some good (e.g. electricity), it also provides an implicit subsidy to a producer’s output where the less emissions-intensive inputs receive a net subsidy (Fischer, 2001; Helfand, 1991; Holland et al., 2009). Generally, a TPS results in inefficient outcomes by encouraging abatement efforts via emission rate reduction as opposed to curbing demand.

The typical analysis of the efficiency of a TPS proceeds by characterizing the pollution externality and the ability of a TPS to internalize the marginal external cost of the externality. Substitution towards less emissions-intensive sources as a result of the subsidy endogenously adjusts the levels of the implicit tax and subsidy. Because an emissions intensity standard taxes only the portion of emissions that exceed the standard and subsidizes inputs with emissions below the standard, the policy is considered a second-best option - as opposed to a first-best option - as it fails to address the marginal external damage of the pollution externality. This issue creates a disconnect between the damages caused by emissions and the efficient level of emissions. Cost effectiveness of the policy requires that the marginal abatement cost be equalized across all sources of pollution. This cannot be the case if some polluters are receiving a net subsidy while others are being taxed.

In this paper I focus on better understanding the TPS within a dynamic context.
Much of the literature exploring the TPS is static in nature, and does not consider dynamics or foresight. The closest related work is Fischer and Newell (2008), where technical change and spillovers are explored in a two-stage stylized electricity model. While foresighted decision-making and discounting is represented in this work, the methods are specifically targeted to the research question and do not allow for a close examination of the initial short-run response, the steady-state policy response, and the transition from the first period through the steady-state.

It is important to analyze a TPS in a dynamic framework because many emissions intensive sectors (e.g. electricity) depend on long-lived capital (e.g. generation capacity) that needs to be replaced as it ages. Also, issues related to climate tend to be longer term problems where investment is likely to be an important margin of adjustment. Regarding foresight, agents make investment decisions regarding long-lived capacity that require some degree of foresight and have implications for the future. Expectations about the future have implications for decision making today.

There are potential factors not captured in a static analysis that could provide new insight when considered in a dynamic context. Adjustment costs and their intersection with the foresighted decision-maker are one such factor. Amigues et al. (2013) consider the extraction of a renewable and non-renewable resource, taking into account that the extraction of the renewable source requires capital investment in renewable electricity generating units (EGUs) and payment for adjustment costs (Gould, 1968). Amigues et al. (2013) show that the best strategy is to start to build renewable power early and spread investment over time as optimal investment in renewables is independent from existing fossil sources. Coulomb et al. (2019) study the transition from a dirty fuel (e.g. coal) to a clean fuel (e.g. gas) and renewable source of power under infinite resource stocks, capacity constraints, a carbon budget, and adjustment costs. They confirm the findings of Amigues et al. (2013). In particular, they find that investment in gas can start

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1Static analyses range from general policy comparisons to examination under imperfect competition, emissions leakage, pre-existing tax distortions, technical change, to energy efficiency implications in the following works: Burtraw et al. (2014); Bushnell et al. (2017); Fell et al. (2017); Fischer (2003, 2011); Fischer and Newell (2008); Holland (2012); Li and Shi (2017); Parry and Williams (2011); Paul et al. (2014)
before coal is phased out and that investment in renewables can start before gas or coal are phased out. The reason for this is that the foresighted planner smooths investment over time in order to reduce adjustment costs.

Additionally, consider that the implicit subsidy to low emitting sources of production under a TPS may incentivize firms to retire dirtier sources and invest in cleaner sources. Firms may even decide to retire dirtier sources early and replace the capacity with cleaner alternatives in order to capture the subsidy. With a TPS, as low emitters enter (and high emitters exit), the average baseline emissions rate falls, loosening the constraint on average emissions, causing the price of permits to fall (Fischer, 2001). These characteristics could have important implications for how decision makers deploy capital in a foresighted context compared to policies, such as a carbon tax or emissions cap, that prioritize output reduction as a form of abatement.

Another factor that could provide new insight is the incorporation of discount rates. Discounting is incorporated to represent the assumption that net benefit flows in the future are not as important to agents as net benefits in the present. Higher discount rates encourage agents to shift abatement efforts towards the present. The channel by which this abatement effort primarily occurs, whether by fuel switching or output reduction, could change the abatement cost and social surplus implications of comparative policies within particular regions of the time horizon, such as the steady-state.

In this paper, I explore a TPS under capacity constraints and investment using a stylized equilibrium model. In section 2, I define an analytical model that I use to provide a complete characterization of the initial period and steady-state output responses to the TPS policy. I find that there exists a performance standard that leaves the price of output unchanged from the no-policy case in the post-policy steady-state. Using this performance standard as a reference point, I find that if I set a new standard that is more (less) stringent than this reference point and the new standard binds, the price of output will be higher (lower) and quantity demanded will be lower (higher) in the post-policy steady-state. These intermediate results are important in understanding how the primal and dual variables of the model respond to the TPS and should be used as an aide in
understanding the output from the numerical model in section 3.

In section 3, I use a numerical model to validate the analytical results and to place the model within the context of an electricity sector. The numerical model is also used to explore the costs and benefits of the TPS in comparison to a no-policy scenario as well as that of a period-over-period damage equivalent cap (CAP). Analysis of the numerical output shows that steeper marginal investment cost curves cause investment to be spread further across the time horizon, as expected. I also find that my results agree with existing literature in that the present value of all social surplus that accumulates to the TPS over time is always less than that of the CAP except when there is a zero emitting fuel and the performance standard is set to zero (Fischer, 2001; Holland et al., 2009). Additionally, the difference in present value of total social surplus changes that accumulate to the TPS and CAP over time are small relative to total social surplus changes associated with either policy.

The main result relates to changes in steady-state social surplus. Introducing a rate of time preference results in a steady-state change in social surplus perpetuity value that is greater under a TPS than that of a CAP. This is true for all but the lowest rates of time preference. A purely static analysis could not yield this result due to the lack of a rate of time preference and its interaction with the foresighted decision-maker. In fact, with a discount rate of zero, steady-state change in social surplus value under the CAP is always greater than the TPS. The main result suggests that some classes of future consumers – specifically those who do not stand to benefit from an intergenerational transfer of wealth – may prefer a TPS to a CAP. The literature (Nordhaus, 2007) on social discount rates and climate change tends to focus on the particular rate of discount chosen and determining whether the net present value of a policy is positive or negative, rather than a comparison of different policies. Philosophical arguments propose the use of a lower social discount rate in order to reflect the fact that institutions of society last longer than any one person and therefore must place greater weight on the well-being of posterity. If government has a lower discount rate than the agents responding to policy, and government is responsible for policy choice and agents for policy response, the main
welfare result in this paper suggests a TPS may inevitably be the policy instrument of choice.

In addition to the benefits associated with a TPS compared to other policies noted by Fischer (2019), Popp (2019), and Goulder et al. (2019), the results in this work highlight new benefits to the TPS. The numerical section also highlights many research avenues that could be pursued in further exploration of the TPS in a dynamic setting and, additionally, provides a baseline for what to expect from a more applied model.

This paper proceeds with a discussion of the analytical intuition (section 2) where the initial period and steady-state policy responses are discussed in sections 2.1 and 2.2. Then I turn to the numerical results in section 3 which proceeds with a discussion of the model setup (section 3.1), the scenario definition (section 3.2), the evolution of the model variables through time (section 3.3), and a discussion of abatement cost and social surplus (section 3.4). Concluding remarks are provided in section 4.

2 Intuition

In this section I define an analytical model used to explore the initial and steady-state responses to the TPS policy instrument. Throughout this section, I use language applicable to the electricity sector. However, the discussion is relevant to other industrial sectors that are subject to long-lived capacity, large costs of investment, and capacity constraints. In section 2.1 I lay out a dynamic model and discuss the initial period response analytically. I also define the period-over-period emissions equivalent cap (CAP) and compare the policy response under perfectly inelastic demand. In section 2.2 I build upon the model and assumptions developed in section 2.1 and perform an analysis of the pre- and post-policy steady-states under different elasticity of demand assumptions and linearly increasing marginal investment costs. The results from this section should be used to inform expectations about the numerical model output.
2.1 The initial policy response

While the following model is dynamic, framing the model statically proves useful in understanding how the primal and dual variables in the model respond to the TPS policy instrument. It is also useful to discuss the intuition of a TPS as it compares to a CAP due to its ability to internalize the marginal external costs of the pollution externality at a relatively low cost. This section should highlight some of the key similarities and differences between a TPS and a CAP. Additionally, I discuss how changing the slopes of demand and supply curves can impact model output as well as how adding investment and capacity constraints can impact policy responses. The changes to the primal and dual variables brought about by these policy instruments are important in understanding the implications for costs and benefits associated with a TPS. Costs and benefits of these policies are discussed in later sections. In this section, assumptions are relaxed step-by-step in order to provide a clear understanding of the policy instrument and the drivers of outcomes.

In the simple model to follow, I assume a perfectly competitive electricity sector with two sources of generation \((X_{f,t})\) – a high (H) carbon source and a low (L) carbon source. Combined generation \((\sum_f X_{f,t})\) from the two fuel sources \((f)\) must equate with the level of electricity demanded \((Q_t)\) in a given time period \((t)\) that occurs in discrete intervals. Over time, generation capacity \((K_{f,t})\) is retired at a rate of depreciation \((\delta)\). As capacity is retired, additional megawatts must be added through investment \((I_{f,t})\). Investment occurs contemporaneously with generation in a given period. Each fuel has an associated emissions intensity \((\omega_f)\) which determines the rate at which emissions are produced from generating electricity.

These characteristics of the electricity sector are represented in the maximization
problem below:

\[
\max_{X,I,Q} \sum_t \left[ \int_0^{Q_t} P_t(Q_t) dQ_t - \sum_f C_{f,t}^I(I_{f,t}) - \sum_f C_{f,t}^X(X_{f,t}) \right] \tag{1}
\]

subject to:

\[
\sum_f X_{f,t} \geq Q_t \tag{2}
\]

\[
K_{f,t} \geq X_{f,t} \tag{3}
\]

\[
K_{f,t-1} \cdot (1 - \delta) + I_{f,t} \geq K_{f,t} \tag{4}
\]

\((CAP \ case)\) \quad \Omega_t \geq \sum_f \omega_f X_{f,t} \tag{5}

\((TPS \ case)\) \quad \sigma_t \geq \frac{\sum_f \omega_f X_{f,t}}{\sum_f X_{f,t}} \tag{6}

\]

\[
X_{f,t}, I_{f,t}, Q_t \geq 0 \tag{7}
\]

where \(X_{f,t}, I_{f,t},\) and \(K_{f,t}\) represent generation, investment, and capacity in period \(t\) for fuel type \(f\), respectively. Equation 1 represents the sum of net benefits across the time horizon. Where benefits \((\int_0^{Q_t} P_t(Q_t) dQ_t)\) are a function of quantity demanded \((Q_t)\) which equals generation summed across all fuel sources in equation 2. \(C_{f,t}^I(I_{f,t})\) and \(C_{f,t}^X(X_{f,t})\) represent total costs of investment and generation, respectively. Net benefits are maximized in equation 1 by choosing generation and investment. The objective function is constrained by equations 2 - 7 using only one of equations 5 or 6. Equation 3 requires that generation not exceed capacity for each fuel type in a given time period. Equation 4 dictates the evolution of capacity via depreciation and investment in each time period for each fuel type. The depreciation rate \((\delta)\) is constant throughout the time horizon. Equation 5 represents the constraint for the CAP case which equates the aggregate emissions cap \((\Omega_t)\) with emissions being produced from generation. Equation 6 represents the constraint for the TPS which requires the rate of emissions from generation to be less than or equal to some intensity target \((\sigma_t)\). The choice variables are greater than or equal to zero for all fuel types and time periods.

The Lagrangian for this optimization problem is as follows:
\[
\mathcal{L} = \sum_t \left[ \int_0^{Q_t} P_t(Q_t) dQ_t - \sum_f C'_{f,t}(I_{f,t}) - \sum_f C_X^{f,t}(X_{f,t}) \right] \\
- \sum_t PD_t \left( Q_t - \sum_f X_{f,t} \right) \\
- \sum_t PK_{f,t} \left( X_{f,t} - K_{f,t} \right) \\
- \sum_t PI_{f,t} \left( K_{f,t} - K_{f,t-1}(1-\delta) - I_{f,t} \right)
\]

(CAP case) \[- \sum_t PCAP_t \left[ \sum_f \omega_f X_{f,t} - \Omega_t \right] \]

(TPS case) \[- \sum_t PTPS_t \left[ \sum_f \omega_f X_{f,t} - \sigma_t \sum_f X_{f,t} \right] \]

where the rental rate on capacity \((PK_{f,t})\) is associated with the capacity constraint, the price of investment \((PI_{f,t})\) is associated with the equation of motion, and the permit prices are associated with each policy constraint \((PCAP_t\) and \(PTPS_t\)). The first order conditions for each case with respect to \(X_{f,t}\) yield an equation that provides some useful intuition.

\[
\mathcal{L}^{\text{CAP}}_X: \quad PD_t = MC_{f,t}^X + PK_{f,t} + PCAP_t \omega_f 
\]

\[
\mathcal{L}^{\text{TPS}}_X: \quad PD_t = MC_{f,t}^X + PK_{f,t} + PTPS_t (\omega_f - \sigma_t) 
\]

\[
\mathcal{L}^{\text{BAU}}_X: \quad PD_t = MC_{f,t}^X + PK_{f,t} 
\]

Additionally note the following zero profit conditions with respect to \(K_{f,t}\) and \(I_{f,t}\):

\[
\mathcal{L}_K: \quad PK_{f,t} = PI_{f,t} - PI_{f,t+1}(1-\delta) 
\]

\[
\mathcal{L}_I: \quad PI_{f,t} = MC_{f,t}^I 
\]

For now, assume that demand is perfectly inelastic. This means that \(P_t(Q_t) = \overline{p}_t\), where \(\overline{p}_t\) is some positive constant. Figure 1 illustrates in simple terms the behavior of a
CAP or TPS as it compares to business-as-usual (BAU) under perfectly inelastic demand. The case of perfectly inelastic demand is useful in that it allows for examination of policy responses when total generation is fixed, which requires emissions regulations to be met via fuel switching.

Before exploring figure 1 in more detail, further assumptions are needed. For simplicity, assume that marginal costs of generation \( MC_{X,f,t} \) and investment \( MC_{I,f,t} \) are constant and that both fuels are operating at their physical capacities to generate in the BAU state. Further assume that \( MC_{X,H,t} = MC_{X,L,t} \) and \( MC_{I,H,t} = MC_{I,L,t} \). At this point, assuming that the emissions intensity for the high carbon fuel is higher than the emissions intensity for the low carbon fuel \( \omega_H > \omega_L > 0 \) guarantees that under enforcement of either policy constraint (equation 5 or 6) coal will be the marginal fuel in production. In order to assess the policy instrument immediately upon enforcement, assume that the time horizon is a single period. Lastly, assume that the BAU case establishes benchmark levels for the average emissions intensity and the aggregate level of emissions, \( \Phi_t \) and \( \Psi_t \) respectively. \( \Phi_t \) and \( \Psi_t \) are related by generation in the BAU state such that \( \Psi_t = \Phi_t \sum_f X_{BAU}^{f,t} \). These benchmark values are then reduced by a rate of emissions reduction \( \rho_t \) to establish the policy targets such that \( \Omega_t = (1 - \rho_t) \Psi_t \) and \( \sigma_t = (1 - \rho_t) \Phi_t \). This specification of \( \Omega_t \) and \( \sigma_t \) defines the CAP and TPS policy cases respectively.

Returning to figure 1 note that two states of the world exist, the BAU state and the CAP/TPS state. Given the assumptions above, figure 1 illustrates how a CAP and TPS are equivalent policies in quantity space when quantity demanded is fixed. Total emissions and total generation are represented by the left and right hand sides of figure 1 respectively. Quantity demanded is represented by the line segment \( FI \). Quantity demanded is met by generation from H or L which are stacked in the columns on the right side of the diagram. In the BAU case, H produces half of the electricity and L produces half of the electricity, shown by the fact that line segment \( FG = GI \). In order to meet demand, electricity is generated, which produces emissions of CO2. Under the BAU case, aggregate emissions are given by line segment \( AE \). Again, emissions attributed to each fuel source are stacked in the columns on the left side of the diagram. Recall that
the emissions intensity for H is greater than the emissions intensity for L ($\omega_H > \omega_L$). Given that H and L contribute equally to the generation mix in the BAU case, it follows that H produces a larger share of total emissions than that of L. This is represented by the fact that $AC < CE$ in figure 1.

The CAP/TPS case imposes a policy that will reduce the average or aggregate emissions rate by the same percentage. This in combination with perfectly inelastic demand will lead to an equivalent reduction in aggregate emissions (given by segment $AB$) regardless of whether it is the rate (6) or level (5) of emissions that is regulated. This is because output of electricity will not change. Recall that H and L are operating at their physical capacities to generate in the BAU state. When emissions are reduced via a CAP or TPS, perfectly inelastic demand prevents consumers from abating emissions by reducing consumption of electricity. Abatement of emissions must occur via fuel switching. In order to reduce emissions via fuel switching, L must increase its share of generation and H must decrease its share in order to dilute aggregate emissions via relative differences in emissions intensity. In figure 1, the switch from H to L is given by the line segment $GH$. Building new capacity in L for generation and leaving excess H capacity leads to a generation mix that is heavier in L production than in the BAU case. The new generation mix results in an emissions mix diluted by L relative to BAU. The result is represented in figure 1 by the fact that emissions from L make up a higher share of the emissions mix in the CAP/TPS case than the BAU case (given by $BD/BE > AC/AE$). L capacity is constrained, thus more L capacity needs to be built in order to satisfy the policy. On the other hand, producers substitute away from H, leaving excess (unused) H capacity in the sector. This is how emissions can be reduced via fuel switching alone, which regardless of policy choice is the only abatement option given our assumptions.

In the case of perfectly inelastic demand total generation must meet the fixed level of quantity demanded such that:

$$\sum_f X_{f,t}^{BAU} = \sum_f X_{f,t}^{TPS} = \sum_f X_{f,t}^{CAP} = d_t \quad (14)$$
where $\sum_f X_{f,t}^{TPS}$ and $\sum_f X_{f,t}^{CAP}$ represent total generation after enforcing the TPS or CAP, respectively. Under perfectly inelastic demand, there is effectively no difference in quantity response between a CAP and TPS using the same rate of emissions reduction ($\rho$) to establish each policy target. Recall that $\Phi_t$ and $\Psi_t$ are related by generation in the BAU state such that:

$$\Psi_t = \Phi_t \sum_f X_{f,t}^{BAU}$$

The benchmark values are then reduced by a rate of emissions reduction ($\rho$) to establish each respective policy target such that:

$$(CAP) \quad \Omega_t = (1 - \rho)\Psi_t$$

$$(TPS) \quad \sigma_t = (1 - \rho)\Phi_t$$

To formalize why establishing each policy target in such a way under perfectly inelastic demand results in emissions equivalent policy, I specify the CAP policy target ($\Omega_t$) that results in the same level of annual emissions as a TPS ($\sigma_t \sum_f X_{f,t}^{TPS}$). Given that the TPS is specified according to equation 17, an emissions equivalent CAP is specified as follows:

$$\Omega_t = \sigma_t \sum_f X_{f,t}^{TPS} = (1 - \rho)\Phi_t \sum_f X_{f,t}^{TPS} = (1 - \rho)\Psi_t \sum_f X_{f,t}^{TPS} \sum_f X_{f,t}^{BAU}$$

$$\Omega_t = \sigma_t \sum_f X_{f,t}^{TPS} = \sum_f X_{f,t}^{TPS} \omega_f$$

This specification in equation 19 is used throughout the paper in reference to the CAP policy instrument. Substituting equation 14 into equation 18 reduces to

$$\Omega_t = \sigma_t \sum_f X_{f,t}^{BAU} = (1 - \rho)\Psi_t$$

which corresponds to equation 16.

While figure 1 provides a high-level summary of how quantities can change under these policies, there are other factors at play. Prices are changing as well. Marginal policy costs ($PC_{f,t}$) are represented by the last term in equations 9 and 10 such that
\[ PC_{f,t}^{CAP} \equiv PCAP_t (\omega_f) \quad \text{and} \quad PC_{f,t}^{TPS} \equiv PTPS_t (\omega_f - \sigma_t). \] The CAP places a price on emissions from both L and H. This tax on all sources of generation will increase electricity prices via equation 9. Equations 9 and 10 show a larger marginal policy cost associated with the fuel with the higher emissions intensity. Given our other assumptions, this ensures that all production of electricity from H is abated prior to any L. In a situation such as figure 1 where demand is perfectly inelastic, the CAP and the TPS will bind their emissions constraints (equation 5 and 6) to the same degree, producing identical emissions prices \((PCAP = PTPS)\) but different marginal policy costs and electricity price by equations 9 and 10.

In the TPS, the implicit subsidy drives the difference in marginal policy costs. If we assume that \(PK_{f,t}\) is fixed, electricity price will increase or decrease by the marginal policy cost associated with each fuel type and policy instrument. However, \(PD_t\) cannot scale with permit price because the marginal policy costs are different for each fuel type. An adjustment in the capacity rental rate \((PK_{f,t})\) is required in order to reconcile the differences between fuel types such that equations 5 and 6 hold. Permit prices are reflective of the fact that H produces only the amount at which it can equate rental prices to operating profits. When the capacity constraint no longer binds for H, as exhibited in figure 1, the rental rate for H falls towards its lower bound at 0, where it is no longer considered scarce. Due to the fact that marginal investment costs are constant for each fuel \((\partial MC^f / \partial I_f = 0)\) and complementary slackness associated with equation 3 holds, the capacity rental rate \((PK_f)\) must be either positive and constant, or 0. This is supported by equations 12 and 13. We have also assumed that marginal operating costs are constant and equal across fuel types. This means that in each period prices equilibrate such that

\[ \Delta PD = \Delta PK_f + \Delta PC_f \quad (21) \]

where \(\Delta\) is the difference between the policy (CAP or TPS) case and the BAU (e.g. \(\Delta PD = PD^{TPS} - PD^{BAU}\)). Equation 21 is obtained by subtracting equation 11 from either of equations 9 or 10 and dropping the \(t\) index. Permit prices scale to the point where it is more cost effective to build a marginal unit of L than to continue producing a
marginal unit of electricity with existing H. L will bind its capacity constraint. Equation 21 is used throughout this paper to explain the behavior of all variables in this model.

Further examination of equations 9 - 13 as they relate to L shows positive marginal policy costs for the CAP ($\Delta PC_{L}^{CAP} > 0$), negative marginal policy costs for the TPS ($\Delta PC_{L}^{TPS} < 0$), and zero change in the capacity rental rate for both cases ($\Delta PK_{L} = 0$). Given our assumptions, this implies that electricity price is higher under the CAP case ($\Delta PD_{CAP}^{L} > 0$) and lower under the TPS case ($\Delta PD_{TPS}^{L} < 0$) via equation 21.

Consider further, a situation where marginal investment costs are no longer constant but increasing in $I_{f,t}$. Increasing marginal investment costs can capture scarcity in capital and labor in the broader market (external adjustment costs) and scarcity in labor that is reallocated away from production and towards investment (internal adjustment costs). Due to the large costs of investment associated with the electricity sector, it is reasonable to assume that adjustment costs exist in some capacity for fossil fuels. While I abstract from renewable fuels in this analysis, increasing costs of investment can be used to represent renewable resource extraction and scarcity (Coulomb et al., 2019) as well as other associated infrastructure challenges (e.g. transmission capacity). As $I_{f,t}$ increases within the period, marginal investment costs are equated to investment prices ($PI_{f,t}$) which will increase. As $PI_{f,t}$ increases, the rental rate of capital ($PK_{f,t}$) increases by equation 12. This becomes more important in later sections when I relax the assumption of perfectly inelastic demand, as this influences the level of output, emissions, and damages associated with a TPS. As shown in equations 9 and 10, electricity prices must be high enough to cover marginal operating, policy, and capacity rental costs in equilibrium. This intuition is expanded upon in section 2.2.

2.2 The steady-state policy response

In the previous section I discussed the initial response to the TPS policy under perfectly inelastic demand. In this section I build upon the model and assumptions developed in the previous section. I perform an analysis of the pre- and post-policy steady-state under different elasticity of demand assumptions and linearly increasing marginal investment
costs. The pre-policy steady-state is the BAU steady-state. The BAU steady-state is one in which all primal and dual variables are constant in perpetuity. I establish an analytical framework for predicting steady-state outcomes under a TPS. This framework accounts for changes in all primal and dual variables and gives insight into steady-state periodic social surplus changes relative to the BAU scenario.

Climate change is a long-term problem, therefore the steady-state outcomes are important to take into consideration as social surplus accrues perpetually. It is also important to know where a policy will eventually lead us after planners have had ample time to respond to a policy. Emissions levels are a focus of the steady-state analysis because marginal damages from emissions are assumed to be constant. Therefore, emissions levels are a direct indicator of damages.

The primary intuitive result is that there exists a TPS policy target that does not change electricity price or quantity demanded in the steady-state. In order for this to be true, the standard must be set such that changes in capacity rental costs and marginal operating costs directly offset changes in marginal policy costs (Section 2.2.1). If the standard is more (less) stringent than this standard and the standard binds, electricity price will be higher (lower) and quantity demanded will be lower (higher) in the post-policy steady-state (Section 2.2.2). Emissions can be inferred from the quantity response. Social surplus changes inclusive of avoided damages can then be calculated.

2.2.1 Establishing a starting point for further analysis

In this section I set up the steady-state investment problem and show how to establish a TPS policy target (σ) that does not change electricity price or quantity demanded in the steady-state, regardless of the prevailing elasticity of demand. It is important to establish this policy target as a starting point for analysis in the following section. The assumptions and model setup developed in this section serve as a base with which to explore alternative TPS policy targets and specifications of demand elasticity in section 2.2.2.

Recall the first order conditions from our model developed in section 2.1. Assume
that in a steady-state, the capacity constraint (3) and the policy constraint (6) bind in perpetuity. Note that in the steady-state all variables are unchanged across time, allowing us to drop the $t$ index. The following first order conditions prove useful for the analysis:

\[\mathcal{L}_{PD} : \quad Q = \sum_{f} X_{f} \quad (22)\]

\[\mathcal{L}_{PK} : \quad K_{f} = X_{f} \quad (23)\]

\[\mathcal{L}_{PI} : \quad I_{f} = K_{f} - K_{f}(1 - \delta) \quad (24)\]

\[\mathcal{L}_{PTPS} : \quad \sigma = \frac{\sum_{f} \omega_{f} X_{f}}{\sum_{f} X_{f}} \quad (25)\]

\[\mathcal{L}_{Q} : \quad P = PD \quad (26)\]

\[\mathcal{L}_{X} : \quad PD = MC^X_f(X_f) + PK_f + PTPS(\omega_f - \sigma) \quad (27)\]

\[\mathcal{L}_{K} : \quad PK_f = PI_f - PI_f(1 - \delta) \quad (28)\]

\[\mathcal{L}_{I} : \quad PI_f = MC^I_f(I_f) \quad (29)\]

I will use these first order conditions (22 - 29) to derive a reduced form of the model. Substituting (29) into (28) yields:

\[PK_f = MC^I_f(I_f)\delta \quad (30)\]

Substituting (30) into (27) yields:

\[PD = MC^X_f(X_f) + MC^I_f(I_f)\delta + PTPS(\omega_f - \sigma) \quad (31)\]

Further substitution of (23) and (24) into (31) yields the reduced form:

\[PD = MC^X_f(K_f) + MC^I_f(K_f)\delta + PTPS(\omega_f - \sigma) \quad (32)\]

where $MC^X_f$ is constant. Equation 32 shows how marginal investment costs and marginal policy costs determine the electricity price in a steady-state where the only investment is the replacement of steady-state capacity. Noting equation 30 and expanding upon it,
the capacity rental rate \((PK_f)\) in the steady state is given by:

\[
PK_f = MC^I_f(I_f)\delta = MC^I_f(K_f\delta)\delta
\]  
(33)

At this point I will establish the TPS policy target at which electricity price and total quantity remains unchanged in the post-policy steady state. The first step is to solve for the policy target \((\sigma)\) that leaves electricity price unchanged. I begin by differentiating 27 which yields the following:

\[
dPD = dPK_f + dPTPS(\omega_f - \sigma)
\]  
(34)

where a variable prefixed with a \(d\) represents a change in the variable from the BAU to the TPS steady-state as a result of the policy. I am trying to solve for the policy target \((\sigma)\) that results in zero change in electricity price \((dPD = 0)\). Setting the left hand side of equation 34 equal to 0 and solving for \(\sigma\) yields,

\[
\sigma_0 = \frac{dPK_f}{dPTPS} + \omega_f
\]  
(35)

Where \(\sigma_0\) corresponds to the policy standard that ensures \(dPD = 0\). Assuming that only a high carbon \((H)\) and low carbon \((L)\) fuel exist I can set the policy target equal for each fuel:

\[
\sigma_0^L = \sigma_0^H
\]  
(36)

\[
\frac{dPK_L}{dPTPS} + \omega_L = \frac{dPK_H}{dPTPS} + \omega_H
\]  
(37)

solving (37) for \(dPTPS\) yields:

\[
dPTPS = \frac{dPK_L - dPK_H}{\omega_H - \omega_L}
\]  
(38)
Substituting (38) into (35) results in the following:

\[ \sigma_0 = \frac{dPK_f(\omega_H - \omega_L)}{dPK_L - dPK_H} + \omega_f \]  

Equation 39 is a general form of the TPS target that results in \( dPD = 0 \). Recall that marginal policy costs are given by \( PC \equiv PTPS(\omega_f - \sigma) \). This equation specifies the policy target \( (\sigma_0) \) that will ensure that equation 21 is equal to 0 as follows:

\[ dPD = dPK + dPC = 0 \]  

To fully appreciate equation 39 it is necessary to consider how the marginal investment cost function drives \( dPK \) and hence \( \sigma_0 \). First, I reiterate the assumptions that demand can take on any elasticity and the marginal investment cost curves are increasing in investment \( (\partial MC^I_f(I_f)/\partial I_f > 0) \). The relationship between the capacity rental rate and the marginal investment cost curves in the steady-state follows from equation 33 such that:

\[ dPK_f = \delta dMC^I_f(I_f) = \delta dMC^I_f(K_f \delta) \]  

In the steady-state, the change in the capacity rental rate is the change in the marginal investment cost multiplied by the depreciation rate. By assuming that the policy target binds \( (\sigma < \Phi) \), it follows from equation 41 that \( dPK_L > 0, dPK_H < 0, dMC^I_L > 0, \) and \( dMC^I_H < 0 \). Finally, assuming that the marginal investment cost curves are linear means:

\[ dMC^I_L(I_L) = \frac{\partial MC^I_L(I_L)}{\partial I_L} dI_L = \gamma_L dI_L = \Delta MC^I_L(I_L) \]  

\[ dMC^I_H(I_H) = \frac{\partial MC^I_H(I_H)}{\partial I_H} dI_H = \gamma_H dI_H = \Delta MC^I_H(I_H) \]

where \( \gamma_f \) is some constant marginal cost. Substituting 41 into 39 yields:

\[ \sigma_0 = \frac{\delta dMC^I_f(\omega_H - \omega_L)}{\delta (dMC^I_L - dMC^I_H)} + \omega_f \]
Cancelling the $\delta$s and substituting (42) and (43) results in the value of $\sigma_0$ specified in terms of the characteristics of the linear marginal investment cost functions, given below:

$$\sigma_0 = \omega_L + \frac{\gamma_L dI_L (\omega_H - \omega_L)}{\gamma_L dI_L - \gamma_H dI_H}$$

Equation 45 establishes a relationship between the relative slopes of the marginal cost functions for each fuel and the placement of a standard that will result in $dPD = 0$. In order for $PD$ to remain unchanged, $Q = \sum_f X_f = \sum_f K_f = \sum_f I_f/\delta$ needs to remain unchanged. This implies that it is always the case that $dI_H = -dI_L$. Given this fact, equation 45 reduces to:

$$\sigma_0 = \omega_L + \frac{\gamma_L (\omega_H - \omega_L)}{\gamma_L + \gamma_H}$$

(46)

Consider the case where the marginal investment cost functions for the high and low carbon fuels are linear and of the same slope ($\gamma_L = \gamma_H$). In this case, equation (46) reduces to,

$$\sigma_0 = \omega_L + \frac{\omega_H - \omega_L}{2} = \omega_H - \frac{\omega_H - \omega_L}{2}$$

(47)

which places $\sigma_0$ at the midpoint between $\omega_H$ and $\omega_L$. Changes in $PK_H$ and $PK_L$ also need to directly offset $PC_H$ and $PC_L$ as $PTPS$ scales with equation 25, respectively.

**Lemma 1.** Under the assumptions to this point, given an initial interior solution, there exists a standard ($\sigma_0$) that does not change electricity price or quantity demanded in the post-policy steady-state.

Lemma 1 provides a useful starting point for examining the steady-state policy responses. The exercise in section 2.2.2 conceptualizes Lemma 1 and then uses it as a benchmark from which to perform different counterfactual experiments.

### 2.2.2 Counterfactual experiments using a simple example

In this section I set up a simple example to help illustrate the steady-state response to a TPS set at different positions relative to $\sigma_0$. I explore how different demand elasticities impact outcomes when the standard is not set at $\sigma_0$. I find that if the standard is more
(less) stringent than $\sigma_0$ and the standard binds, electricity price will be higher (lower) and quantity demanded will be lower (higher) in the post-policy steady-state.

To begin the analysis I set up a simple example and reiterate some important assumptions. For the moment, assume that demand can take on any elasticity between 0 (perfectly inelastic) and negative infinity (perfectly elastic). Figure 2 illustrates the initial BAU steady-state. The BAU steady state is defined by linearly increasing marginal investment cost curves of identical slope. Marginal investment cost for the low carbon fuel is greater than marginal investment cost for the high carbon fuel. Steady-state investment in the low carbon fuel is half of steady-state investment from the high carbon fuel (i.e. $2I_L^{BAU} = I_H^{BAU}$). Steady-state price of investment for the low carbon fuel is equal to that of the high carbon fuel.

Given the example setup in figure 2, we can calculate $\sigma_0$ using equation 45. As previously discussed, due to the fact that $\frac{\partial \delta MC_H}{\partial I_H} / \frac{\partial \delta MC_L}{\partial I_L} = 1$, $\sigma_0$ reduces to equation 47. Additionally, setting the policy target within the system of equations 23 - 29 at this level results in a binding policy constraint such that $\sigma = \sigma_0 < \Phi$. Upon enforcement of this policy, a new steady-state is eventually reached where the following results:

$$\Delta PC_H / \Delta PC_L = -1$$
$$\Delta PK_H / \Delta PK_L = -1$$
$$\Delta PD = PD^{TPS} - PD^{BAU} = 0$$

$\Delta$ is the difference between the TPS steady-state variable and the BAU steady-state variable. In this section, I use $\Delta PC_f$ and $PC_f$ interchangeably, as marginal policy costs are always zero in the BAU steady-state.

In order to visually illustrate the policy response in this example, I turn attention to figure 3. Figure 3 shows features related to the policy target (right panel) and policy response (left panel). The interaction of the features in the left and right panels of figure 3 show how the capacity rental rate and associated policy costs respond to a given policy target, respectively. Taken together, the sum of $\Delta PK_f$ and $PC_f$ result in the change in
the price of output ($\Delta PD$) as a result of the TPS.

Before discussing how to use the graphical analytical tool depicted in figure 3 to perform counterfactual analysis, I discuss the features of the tool. The right panel of figure 3 relates to the policy target, which is associated with $\sigma$. The origin denoted by a 0 in the right panel is associated with the policy target as well. Note the horizontal ($PC_L$) and vertical ($PC_H$) axes associated with the policy target origin. The feasible set of values that marginal policy costs can take on lies to the left ($PC_L \leq 0$) and above ($PC_H \geq 0$) the origin, as L always receives a subsidy and H is always taxed. The line emanating from the origin, referred to from this point forward as the “policy target line”, represents the relative marginal policy costs, which is dictated by the placement of the policy target ($\sigma$). Each policy target line is associated with a specific policy target choice. In the case of figure 3, the slope is equal to 1, indicating that $\sigma_0$ is the midpoint between $\omega_L$ and $\omega_H$. The line $BD$ represents the feasible combinations of marginal policy costs ($PC_f$), given a binding policy constraint associated with $PTPS > 0$. The policy target line intersects line $BD$ at point $C$. Point $C$ establishes the marginal policy costs ($PC_L$ and $PC_H$) associated with a given policy target choice.

The left panel of figure 3 relates to the policy response. Responses to policy arise as a result of changes in investment levels, which cause changes in the capacity rental rate ($\Delta PK_f$) when marginal investment costs are not constant. The origin, denoted by a 0 in the top left of the panel, is associated with the features of figure 3 that relate to the policy response. The feasible set of values that $\Delta PK$ can take lies in the space to the right of ($\Delta PK_L \geq 0$) and below ($\Delta PK_H \leq 0$) the origin, as L will always see increases in investment and H will always see decreases as a result of the TPS. The line emanating from the origin, referred to from this point forward as the “policy response line”, represents the relative changes in the capacity rental rate ($\Delta PK_H / \Delta PK_L$) associated with a given policy target choice, marginal investment cost function, and elasticity of demand assumption. The line $GI$ represents the set of feasible combinations of capacity rental rate responses to the binding policy constraint associated with $PTPS > 0$. The intersection of the policy response line and $GI$ occurs at point $H$. Point $H$ represents the optimal response...
(\Delta PK_L \text{ and } \Delta PK_H) \text{ to the TPS given a policy target choice, marginal investment cost functions for L and H, and an elasticity of demand assumption.}

As in figure 3, if the policy target is set such that \( \sigma = \sigma_0 \), then the policy target line and the policy response line will have the same slope (\( \frac{\Delta PK_H}{\Delta PK_L} = \frac{PC_H}{PC_L} \)). Recalling equation \ref{eq:21}, note that in figure 3 the marginal policy cost and change in capacity rental rate sum to 0 for a given fuel, such that \( \Delta PD = \Delta PK_f + \Delta PC_f = 0 \). The dividing double line (\( KM \)) in figure 3 links the policy response and policy target. For example, in figure 3, \( \Delta PK_H + PC_H = 0 = \Delta PD \) is shown by the intersection of \( KM \) and \( EJ \) at point \( L \).

The following analysis proceeds in phases. Phase 0 is represented in figure 3 and 4. Phase 0 serves as a benchmark from which counterfactual experiments in phases 1 and 2 are performed. An important assumption I make in the counterfactual analysis is that once Phase 0 establishes a level of \( PTPS \), I hold fixed \( PTPS \) at the level established in Phase 0. This is done in order to improve tractability of the analytical model without losing the core intuition by preventing lines \( BD \) and \( GI \) from shifting. Phase 1 explores the case of perfectly inelastic demand and a stricter policy target set such that \( \sigma < \sigma_0 \). Phase 2 explores the case of perfectly elastic demand under the stricter policy target established in phase 1. Together, phases 1 and 2 establish the range of possible outcomes under different demand elasticities.

Before continuing to phases 1 and 2, it is necessary to explore how the phase 0 intensity map impacts output in figure 4. Figure 4 shows electricity price as a function of total capacity (left panel) and investment price as it relates to investment (right panel). Figure 4 shows price and quantity responses for the policy target established in figure 3. Figure 4 reiterates the fact that the magnitude of marginal policy costs are equal and the slopes of the investment cost curves are equal. Changes in investment levels and investment prices for the low and high carbon fuel perfectly offset. This translates into perfectly offsetting changes in capacity and capacity rental rate, resulting in a zero change in total capacity. Marginal policy costs exactly offset changes in capacity rental rates, resulting in a zero change in electricity price. A zero change in electricity price corresponds to a
zero change in total capacity. A summary of the phase 0 result is below:

\[ \Delta PD = \Delta PK_f + \Delta PC_f = 0 \]  \hspace{1cm} (48)
\[ \Delta PC_H/\Delta PC_L = (\omega_H - \sigma)/(\omega_L - \sigma) = -1 \]  \hspace{1cm} (49)
\[ \Delta PK_H/\Delta PK_L = \Delta PI_H/\Delta PI_L = -1 \]  \hspace{1cm} (50)
\[ \Delta I_H + \Delta I_L = 0 \]  \hspace{1cm} (51)
\[ \Delta K_H + \Delta K_L = 0 \implies \Delta \sum_{f} X_f = 0 \]  \hspace{1cm} (52)

Figures 5 and 6 are associated with phase 1. In figure 5, any reference to “1” indicates an element associated with phase 1. Phase 1 shows the response to a stricter standard \((\sigma < \sigma_0)\) under perfectly inelastic demand. The stricter policy target is represented by a rotation of the policy target line clockwise, effectively making the slope of the policy target line steeper. By placing the standard closer to the low carbon fuel, the high carbon fuel bears a higher burden of the cost of compliance. Hence, marginal policy costs for the high carbon fuel are greater in magnitude than the low carbon fuel. This is represented by the intersection of the new policy target line with \(BD\) at point \(N\). The result is that \(PC^1_H > PC^1_L\) and \(PC^1_f > PC^0_f\). Turning attention to the policy response, perfectly inelastic demand prevents deviation from phase 0 (noting that \(PTPS\) is assumed fixed). Total capacity will remain unchanged due to perfectly inelastic demand. This means that electricity price must increase in order to ensure that \(\Delta PD = \Delta PK_f + \Delta PC_f\).

Turning to figure 6, it can be seen how changing relative marginal policy costs via the policy target relative to phase 0 impacts quantities under perfectly inelastic demand. As expected, quantities do not change relative to phase 0 under perfectly inelastic demand. An increase in electricity price is the only way to balance the phase 1 marginal policy costs and capacity rental rates associated with a zero change in total capacity, as depicted...
in the left panel of figure 6. See below for a summary of results from phase 1:

\[ \Delta PD = \Delta PK_f + \Delta PC_f > 0 \]  
(53)

\[ \frac{\Delta PC_H}{\Delta PC_L} = \frac{\omega_H - \sigma}{\omega_L - \sigma} < -1 \]  
(54)

\[ \frac{\Delta PK_H}{\Delta PK_L} = \frac{\Delta PI_H}{\Delta PI_L} = -1 \]  
(55)

\[ \Delta I_H + \Delta I_L = 0 \]  
(56)

\[ \Delta K_H + \Delta K_L = 0 \implies \Delta \sum_f X_f = 0 \]  
(57)

Phase 2, given in figures 7 and 8, assumes the same adjustment to the policy target line as phase 1 but allows for perfectly elastic demand. This effectively allows for adjustments in total capacity in order to maintain the prevailing electricity price, forcing \( \Delta PD = 0 \). Again, any reference to a “2” in figure 7 indicates an element associated with phase 2. First, the rotation of the policy target line in phase 2 (figure 7) is identical to the rotation of the policy target line in phase 1 (figure 5). This carries the same implications regarding relative marginal policy costs as phase 1. The difference between figures 7 and 5 is that in figure 7 the policy response line rotates clockwise around the origin. The policy response line rotates until its slope is identical to that of the policy target line. The intersection of the new policy response line and \( GI \) at point \( P \) dictates the changes in capacity rental rates for \( H \) and \( L \). The policy response line rotates until its slope is just equal to that of the policy target line, coinciding with a zero change in electricity price \( (\Delta PD = \Delta PK_f + \Delta PC_f = 0) \).

Figure 8 shows how electricity price and quantities adjust to the stricter policy target under perfectly elastic demand. In order to ensure that electricity price remains unchanged, quantities must respond such that changes in the capacity rental rate for each respective fuel perfectly offset changes to marginal policy costs brought about by the policy. In order for this to be true, investment in the low carbon fuel will increase, but by less than in phase 0. Investment in the high carbon fuel will decrease by a larger amount than in phase 0. As depicted in the right panel of figure 8, investment in the low carbon fuel will increase by less than investment in the high carbon fuel decreases. As a
result, the price associated with investment in the low carbon fuel will increase by less than the investment price associated with the high carbon fuel decreases. This results in a steady-state comprised of a smaller amount of total capacity than in phase 0 (or the BAU steady-state). A summary of results from phase 2 is below:

\[ \Delta PD = \Delta PK_f + \Delta PC_f = 0 \]  
(58)

\[ \Delta PC_H/\Delta PC_L = (\omega_H - \sigma)/(\omega_L - \sigma) < -1 \]  
(59)

\[ \Delta PK_H/\Delta PK_L = \Delta PI_H/\Delta PI_L < -1 \]  
(60)

\[ \Delta PC_H/\Delta PC_L = \Delta PK_H/\Delta PK_L < -1 \]  
(61)

\[ \Delta I_H + \Delta I_L < 0 \]  
(62)

\[ \Delta K_H + \Delta K_L < 0 \implies \Delta \sum_f X_f < 0 \]  
(63)

Phase 1 and phase 2 place boundaries on the possibilities for steady-state outcomes given varying demand elasticities. Any demand elasticity that is not perfectly inelastic will cause a clockwise rotation in the policy response line for any standard that is placed such that \( \sigma < \sigma_0 \). Additionally, not depicted in figure 7 due to the assumption that PTPS is fixed, the policy target frontier will shift towards the origin relative to its phase 1 position. This is due to a loosening of the policy constraint (25) brought about via abatement of consumption. In phase 1, all abatement is achieved through fuel switching via investment. In phase 2, a mix of the two forms of abatement occurs. The analysis above leaves us with the following intuitive result:

**Proposition 1.** If the standard \( (\sigma) \) is more (less) stringent than \( \sigma_0 \) and the standard binds \( (\sigma < \Phi) \), electricity price is higher (lower) and quantity demanded is lower (higher) in the post-policy steady-state.

The intuitive exercise discussed in this section highlights some of the key drivers underlying the TPS and the model construct. Most notably, the policy outcome is sensitive to the placement of the policy target, the relative slopes of the marginal investment cost curves, the degree of difference between the rate at which marginal policy costs and ca-
pacity rental rates scale, and the elasticity of demand. This analytical framework also provides a policy response for all primal and dual variables in the steady-state. This allows for calculation of emissions levels in the post-policy steady-state and further analysis of damages, abatement cost, and social surplus.

This result is not limited to dynamic models as it is static in nature. In fact, due to the assumptions made, $PK$ could simply be represented by an upward sloping marginal cost function of any kind. The analytical results here can be referenced in lieu of a detailed decomposition of the output effects of a TPS in the case of most applied analyses.

3 Numerical Results

In the previous sections I established analytical intuition for the behavior of the dynamic model under the initial and steady-state responses to the TPS policy instrument. In section 3.3, I use a numerical version of the model to examine the evolution of the model variables through time under a TPS. In section 3.4, I discuss the costs and benefits associated with the TPS policy, and compare the TPS results to the BAU scenario and that of an emissions equivalent CAP. Prior to discussing the results, I relay some important details related to the numerical model setup and define the model scenarios in sections 3.1 and 3.2.

3.1 Numerical model setup

Before discussing the scenario definition and model results, it is important to make note of some characteristics that place this model within the context of electricity markets. While electricity markets are the subject of this analysis, a detailed representation is beyond the scope of the study. A highly detailed representation is not necessary for determining the underlying relationship between investment costs, capacity constraints, policy stringency, and time. However, electricity markets are subject to large investment costs and long-lived generation capacity. These features subject the markets to capacity constraints which may be either binding or non-binding. These features are relevant in
many industrial sectors of an economy.

Given the scope of the numerical analysis, it is important to maintain the relationship between costs of generation and investment. Particularly the fact that investment costs are large and that investment produces capacity which can be used over an extended period for the purpose of generating electricity at an additional operating cost. Table 1 provides some of the key parameters used within the numerical model, as well as their values.

The parameters are chosen to loosely represent operating costs relative to investment costs within the electricity sector (EIA, 2018a,b,c; IEA, 2018; NREL, 2018). The value for elasticity of demand was chosen to represent consumer responses to price over the medium to long term (Deryugina et al., 2020). The values for exogenous demand and the depreciation rate were chosen to ease interpretation of the results. Furthermore, the emissions intensity of L relative to H does not have a major impact on the intuition obtained from the numerical results. However, when \( \omega_L = 0 \) it has the added benefit of showing convergence in social surplus between the CAP and TPS as the standard approaches zero. The number of operating hours in each year are included to make sure that costs over each period \((t)\) are aligned in terms of operations and investment. For example, generation must meet demand which is modeled in megawatts \((MW)\). The cost of generating at 100 \(MW\) over the course of a year is \(100 \times MC_{f,t} \times hiy\) in dollars, because 100 megawatts generated over 1 period of 8760 hours is \(100 \times 8760\) megawatt-hours \((MWh)\). If the depreciation rate is 0.05 and the capacity constraint binds, then 100 \(MW\) of capacity must be maintained every period in the steady-state. This means that investment must replace the 5 \(MW\) of depreciated capacity at a high cost (e.g. $1,000,000/MW).

In the numerical model I choose the slope and intercept of the marginal investment cost curves. The intercept term is always positive and identical across fuel types while the slope can vary for the scenarios modeled. I choose the slope and intercept so that the business-as-usual (BAU) steady-state is one in which \(\bar{d}_t\) is constant throughout the time horizon at an initial period marginal price of investment \((PI_{f,t=0})\) equal to $1,000,000/MW. The
value of $1,000,000/MW was chosen to place investment costs in the general vicinity of US electricity sector costs. The slopes are determined by the following function:

\[ PI_{f,t} = \left[\frac{1}{(1 + r)}\right]^t \times [\alpha + \beta_f \times I_{f,t}] \]  

(64)

where \(\alpha\) is the intercept term of the marginal investment cost function and \(\beta_f\) is the slope term. In an example where the slopes are identical (i.e. \(\beta_L = \beta_H\)) the BAU steady-state is such that \(I_{f,t} = \frac{d_t}{2} \times \delta\). In the BAU steady-state, \(d_t = \sum_f X_{f,t} = \sum_f K_{f,t} = 100\) and and \(\delta = 0.05\). The BAU steady-state corresponds to one where capacity is split equally between each fuel such that \(K_{H,t} = K_{L,t} = 50\ MW\) and investment is such that \(I_{f,t} = \delta K_{f,t} = 2.5\ MW\). This is due to the identical intercepts and slopes. Setting \(\alpha = 0\), \(PI_{f,t=0} = 1e6\), and \(I_{f,t} = 2.5\) in equation 64 and solving for the slope yields \(\beta_f = 4e5\).

Once I have the slope and intercept term established, I solve the system of equations with exogenous demand fixed at \(\bar{d}_t\). The marginal value from the market clearance condition equating total generation (\(\sum_f X_{f,t}\)) to quantity demanded (\(\bar{d}_t\)) determines the reference level for the discounted electricity price (\(\bar{pq}_t\)) which falls at a rate of \((1+r)\) over time in the BAU steady-state. The first order condition of the Lagrangian to this problem with respect to the total quantity demanded shows that the benchmark electricity price \(\bar{pq}_t\) is equal to \(\frac{\bar{pd}_t}{(1/(1+r))^t}\). Therefore, \(\bar{pq}_t\) is constant across the entire time horizon.

Using these new values for \(\bar{pq}_t\) and \(\bar{pd}_t\) I allow for price responsive demand where the following equilibrium conditions hold:

\[ \left(\frac{1}{1 + r}\right)^t \times \bar{pq}_t \times \left(\frac{Q_t}{\bar{d}_t}\right)^{1/\varepsilon} \geq PD_t \perp Q_t \]  

(65)

\[ \sum_f X_{f,t} \geq Q_t \perp PD_t \]  

(66)

where \(Q_t\) is total quantity demanded. Solving the model with the updated benchmark values for \(\bar{pq}_t\) and \(\bar{d}_t\) will replicate the BAU steady-state. This provides a basis for comparison with counterfactual policy scenarios allowing for price-responsive demand.

A full set of numerical model equations can be found in appendix A.
3.2 Scenario definition

In order to explore the TPS further, I have decided upon a set of scenarios indicating different model assumptions by varying the slopes of the marginal investment cost curves and policy stringencies. See table 2 for details.

The number (e.g. 1,2) in the scenario name corresponds to the specification of the marginal investment cost functions whereas the letter (e.g. A, B) corresponds to the policy stringency. Scenario S1A corresponds to identical and linear marginal investment cost curves of relatively steep slope with a policy target set at 0.7 times \( \sigma_0 \). The marginal cost curves for each fuel type emanate from the same point for each scenario, therefore \( \sigma_0 \) equals the reference average emissions rate (\( \Phi \)). This means that a standard set at 0.9 times \( \sigma_0 \) corresponds to a 10\% reduction from the reference emissions rate (\( \sigma = \sigma_0 \times 0.7 = \Phi \times (1 - 0.3) \)). Scenario S2A corresponds to identical and linear marginal investment cost curves of relatively flat slope with a policy target set at 0.7 times \( \sigma_0 \). The policy target associated with the highly stringent policy is set at 0.3 times \( \sigma_0 \).

The slope and intercept associated with scenario 1 corresponds to \( \beta_f = 4e5 \) and \( \alpha_f = 0 \). The slope and intercept for scenario 2 corresponds to \( \beta_f = 4e4 \) and \( \alpha_f = 9e5 \). The relatively flat marginal investment cost curves associated with scenario 2 are probably closer to that of the electricity sector than those in scenario 1.

3.3 Evolution of model variables through time

In this section I discuss the evolution of the model variables through time. I discuss the initial period response to the TPS policy followed by a discussion of the transition to the steady-state. I refer back to the theory developed in the previous section where applicable.

Figure 9 shows the initial period response to the TPS policy. Recall that the relationship between electricity price, the capacity rental rate, and the policy cost is defined by the following equation:

\[
\Delta P D = \Delta P K_f + \Delta P C_f
\] (67)
Figure 9 shows that this relationship holds for each scenario and each fuel type (i.e. H and L on the right axis). Marginal policy costs (left column) for H are greater than zero in each scenario. This is associated with a permit price that is greater than zero and an emissions intensity for H that is greater than the intensity standard. The capacity rental rate is 0. Therefore, the change in the capacity rental rate from the BAU (middle column) for H is less than zero for all cases. A capacity rental rate of zero suggests that the capacity constraint does not bind in the initial period for H, as expected. The change in electricity price (right column) is the sum of the change in marginal policy cost and change in capacity rental rate, which is positive.

L sees a negative marginal policy cost, which indicates a net subsidy is received by producers of the low carbon fuel. Changes in the capacity rental rate from the BAU are greater than zero. This indicates that investment changes from the BAU for L are also greater than zero. This increase in investment comes at a higher cost, which is reflected in the capacity rental rate. The sum of the subsidy associated with the marginal policy cost and the increase in the capacity rental rate is equal to the change in the output price from the BAU level, which is positive.

The increase in the output price indicates that total quantity demanded falls, hence total generation falls. The increase in the capacity rental rate for L is associated with an increase in investment for L, which is associated with an increase in capacity to generate. The capacity rental rate would not increase unless the capacity constraint were binding. This means that generation associated with L increases. If total generation decreases, and generation of L increases, then generation of H must decrease. If total generation decreases, then aggregate emissions decrease.

Note in comparing scenarios S1A with S2A, the scenario with the steeper marginal investment cost curves (S1A) results in a higher capacity rental rate and smaller subsidy for L, which results in a higher electricity price. This is due to the fact that responding to policy is more costly when the marginal investment cost curves are steeper. Additionally, a more stringent rate standard results in a larger marginal policy cost for H and smaller subsidy for L given identical cost structures across policy scenarios (e.g. S1A vs. S1B).
The more stringent standard also increases investment required in order to meet it, as depicted by the increased rental rates for L and electricity price when comparing policy case A versus B. All of this intuition can be derived from the results referenced in figure 9.

Total quantity demanded falls in the initial period. This is due to the fact that the slopes of the marginal investment cost curves are steep enough in all scenarios to cause consumers to abate consumption in the initial period. Rather than choosing to invest enough in L to reach the steady-state capacity level immediately, investment is delayed in order to save by spreading costs across multiple periods. This is the result of a foresighted effort to maximize the difference between benefits and costs across the entire time horizon.

Figure 10 shows investment levels relative to the BAU scenario by fuel type and time period. The horizontal black line indicates the BAU steady-state level of investment for each fuel type. Changes can be inferred relative to this line. The first row of figure 10 corresponds to scenarios S1A (left column) and S1B (right column) and the second row corresponds to scenarios S2A and S2B. In comparing the first and second rows, three key observations can be made. First, it is clear that scenario S1A shows lower initial levels of investment in L than S1B. This validates the previous discussion related to the initial policy response. Second, S2A reaches steady-state levels of investment sooner than S1A. This is due to the fact that investment is more costly in S1A and it is therefore more cost effective to consume less and spread the costs across the time horizon. Thirdly, related to the second observation, H investment begins to re-enter in S2A sooner than S1A. This is due in part to the flatter marginal investment cost curves in S2A. Investment in L can be increased at a lower marginal cost in S2A and decreased investment in H does not lower marginal investment costs as much. This skews the investment path in favor of L, allows H to depreciate without investment for longer, and allows H investment to re-establish a new steady-state rental rate faster at relatively low marginal costs.

In comparing the less stringent policy (left column) and the more stringent policy (right column) of figure 10 it is clear that more investment is made in L in order to satisfy the policy initially and across the entire time horizon. This shows that an unexpected
immediate increase in policy stringency can have significant implications on investment levels and costliness of the TPS initially and through the transition period. While not explicitly modeled, these results highlight the intuition that there may be benefits to allowing time to plan ahead for the TPS policy.

Figure 11 shows the capacity utilization rate by scenario with scenarios grouped in columns by the policy stringency. Figure 11 supports the intuition above. It shows full capacity utilization is reached sooner in the scenarios associated with the relatively flat marginal investment cost functions (i.e. S2A and S2B). This is a requirement for reaching the steady-state. This figure also highlights the differences between policy stringencies. The stricter policy takes longer for the capacity constraint to bind for H and to reach a steady-state as investment is smoothed over time in order to reduce overall costs.

Figure 12 shows the percentage change in emissions from the BAU case for each TPS scenario grouped by policy stringency. As expected, the more stringent policy target associated with column B shows a larger reduction in aggregate emissions levels. Additionally, S1A shows larger emissions reductions than S2A due to steeper marginal investment cost curves associated with S1A. In short, more damages are avoided due to the higher costs of investment associated with the steeper marginal investment cost curves. Consumers prefer to reduce more consumption at higher marginal costs of investment. The capacity rental rate increases as a result of higher marginal investment costs which outstrip the net subsidy for production of L. This increases the price of output and decreases quantity produced referenced in equation 67. Reducing consumption comes with the added benefit of avoiding damages from emitting. This result has important implications in assessing the damages avoided as a result of the TPS, an important element of the social surplus calculation.

Figure 13 shows the steady-state period-over-period changes in marginal policy cost ($\Delta PC_f$), changes in capacity rental rate ($\Delta PK_f$), and changes in output price ($\Delta PD$) in each column from left to right. The top row is associated with H and the bottom row is associated with L. As a discount rate of 1% was used in this model, the price output from the model is adjusted to a future value in order to view the steady-state in terms of
perpetually constant values instead of values perpetually decreasing at the rate of \( \frac{1}{1+r} \). Regardless of discounting, the values are of the same proportion to one another.

The results in figure 13 validate the theory developed in section 2.2. The emissions rate standard \((\sigma)\) is chosen such that \(\sigma < \sigma_0\), where the definition of \(\sigma_0\) is established in relation to lemma 1. Since \(\sigma_0\) is equal to the reference period-over-period average emissions intensity in the BAU steady-state such that \(\sigma_0 = \Phi\), the standard is binding. Following from proposition 1, due to the fact that \(\sigma\) is more stringent than \(\sigma_0\) and demand is price responsive, the output price will increase and total quantity demanded will decrease. If quantity demanded decreases, we know from Holland et al. (2009) that emissions levels will decrease as well.

Additionally, steeper marginal investment cost curves result in a greater steady-state reduction in quantity demanded from the BAU. This can be seen by comparing S1A with S2A in the right column of figure 13. As expected, more stringent policy is associated with a higher steady-state electricity price. This is evidenced by comparing S1A with S1B for example.

### 3.4 Abatement cost and social surplus

Turning attention to costs and benefits associated with the TPS policy, I begin by discussing the results in table 3. Table 3 shows present value of changes in the key metrics associated with the social surplus calculation. Please see appendix B for detailed surplus calculations. All values except for the rightmost column represent changes from the BAU value for each metric (i.e. \(\Delta = TPS - BAU\)). The far right column represents social surplus changes relative to an emissions equivalent CAP. Each scenario initially defined in table 2 is listed in the far left column of table 3. The column headings correspond to present value changes in consumer surplus (\(\Delta CS\)), producer surplus (\(\Delta PS\)), investment costs (\(\Delta CInv\)), abatement costs (\(\Delta AC\)), avoided damages (\(\Delta Dam\)), and social surplus.
\( \Delta SS \) summed across time, such that:

\[
\Delta AC = -1 \times (\Delta CS + \Delta PS) + \Delta CInv \tag{68}
\]

\[
\Delta SS = \Delta Dam - \Delta AC \tag{69}
\]

The values themselves are less important than the relationships between scenarios and across the different elements of the surplus calculation. For example, \( \Delta AC \) is larger for S1A than S2A. This is due to the fact that S2A corresponds to the scenario with the flatter marginal investment cost curves. Steeper marginal investment cost curves associated with S1A and S1B result in a larger change in electricity price initially, through the transition, and into the steady-state. This has implications for the magnitude of changes in consumer and producer surplus, the sum of which always represents a positive net cost for the scenarios modeled. Changes in total investment costs will also be larger for S1A and S1B than in S2A and S2B.

Marginal damages are constant across time and all levels of emissions. Therefore, the choice of the value can influence whether or not social surplus changes are positive. For the scenarios modeled, avoided damages exceed abatement costs at a marginal damage level of $20/mt CO2e, indicating positive changes in social surplus relative to the BAU scenario.

The far right column of table 3 shows the difference between changes in social surplus from the TPS relative to the period-over-period emissions equivalent CAP. The CAP always shows greater present value of social surplus changes summed across time than the TPS except for the case where the emissions intensity of L is zero \((\omega_L = 0)\) and the standard is set to zero \((\sigma = 0)\). If \(\omega_L = 0\), as the standard is tightened, the difference between the present value of \(\Delta SS_{TPS}\) and \(\Delta SS_{CAP}\) approaches zero. This movement can be seen in table 3 by comparing scenarios A and B for a given marginal investment cost curve.

Additionally, it is worth noting that the difference in the present value of social surplus changes between a TPS and a CAP is small relative to present value of social surplus
gains from either policy. The surplus differences due to the policy choice itself accounts for roughly 0.74% and 0.03% of $\Delta SS$ under the TPS for scenarios S1A and S2B respectively.

As it relates to abatement cost and social surplus measures, there are two key observations from the numerical output listed below.

**Numerical Result 1.** The present value of all social surplus changes from the benchmark case will always be lower for the TPS than for the period-over-period emissions equivalent CAP, except when $\sigma = 0$ and $\omega_L = 0$.

**Numerical Result 2.** The value of the perpetuity of steady-state social surplus changes can be higher or lower for the TPS relative to the emissions equivalent CAP.

When taken together, numerical results 1 and 2 state that, while the present value of social surplus must always be greater for a period-over-period emissions equivalent CAP, the periodic steady-state future value of social surplus under the TPS may be greater than the CAP. Table 4 shows the ratio of changes in periodic steady-state social surplus under the TPS versus the CAP for a TPS policy target that is more stringent than $\sigma_0$. A value greater (less) than 1 indicates the TPS (CAP) has greater steady-state social surplus change. Table 4 shows that for all but the smallest discount rates, the TPS shows greater steady-state changes in social surplus than the CAP.

The CAP has the ability to respond to policy in a more cost effective manner for any given period. In the case of a zero discount rate, both the CAP and TPS intertemporally optimize across the entire time horizon. The result is an investment path indicative of spreading costs across the time horizon as to minimize costs or maximize net benefits subject to the policy constraint. Under a high rate of time preference, the decision-maker under both policies chooses to shift abatement costs toward the present. As explored in section 3.3, the TPS does this by investing more in the earlier years and abating less via output reduction, whereas the CAP does this by abating more via output reduction in earlier years, allowing the high carbon fuel to depreciate for longer, and delaying investment.

For a low discount rate, the value of the steady-state perpetuity under a CAP will be higher than under a high discount rate. The CAP chooses to forgo perpetual net benefits
at a high discount rate in order to extract net benefits earlier, whereas the TPS increases investment in the low carbon fuel in earlier periods. Under a very low discount rate the value of the perpetuity is so large that it makes more sense to extract those net benefits later while forgoing the cost of abating larger amounts in the early periods. The increased electricity prices that arise from the respective abatement efforts have a negative impact on consumer surplus across the time horizon under the CAP relative to the TPS. Because the CAP can utilize all channels of abatement optimally, it can extract more net benefits in the earlier periods through intertemporal adjustments under higher discount rates than the TPS. A model with a discount rate of zero results in periodic steady-state surplus changes that are always lower for the TPS than the emissions equivalent CAP.

4 Conclusion

In this paper I explore the behavior of a TPS in a dynamic setting. I define an analytical model and discuss the initial and steady-state responses to the TPS policy instrument intuitively. I also use a numerical model to provide additional validation of the analytical results, further explore the transition of the model variables from the initial policy response to the steady-state, and discuss abatement cost and social surplus relative to a no-policy case and a period-over-period emissions equivalent cap.

The intuitive analysis uncovers an emissions intensity standard that does not change the price of electricity in the post-policy steady-state. If the emissions intensity target is set more stringently than the standard that leaves price unchanged, electricity price will rise and quantity demanded will fall. In this case the TPS will result in a lower level of steady-state emissions. The numerical analysis validates this analytical result. Additionally, the numerical model finds that investment levels and abatement costs are sensitive to the slopes of the marginal investment cost curves as well as the policy stringency. Steeper marginal investment cost curves cause investment to be spread out further across the time horizon, as consumers reduce consumption to a greater degree. Abatement costs are also larger in environments with steep marginal investment cost curves.
In comparing the TPS surplus measures to that of a period-over-period damage equivalent CAP, the present value of social surplus changes from the no-policy case is smaller for the TPS than the CAP, though not by much. However, introducing a discount rate leads to a situation where the value of the steady-state social surplus perpetuity is larger for the TPS than for the CAP. This suggests that some classes of future consumers may prefer a TPS to a CAP. Finally, as expected, if the TPS policy target is set to an emissions intensity of zero at the outset, there is no difference in the present value of social surplus between the TPS and CAP across time, regardless of the discount rate.

Future research could explore the ability for decision makers to plan for expected policy with and without policy uncertainty. Different representations of capital accumulation and adjustment could be a valuable addition to this research. Alternative methods for specifying the TPS and comparison policies across the time horizon as well as methods for use of permit or tax revenue from a cap and trade system or carbon tax (i.e. banking and borrowing of permits). Other research could surround policy implementation in a dynamic context. For example, given a carbon budget, determine the best way to set the TPS policy target. Furthermore, other factors such as technology cost improvements could be explored. Applied models that account for capital adjustment, price responsive demand, and foresight can use these results to inform expectations about model output. An applied model that includes these aspects could provide more reliable projections related to a specific industry in question. A final aspect that this work does not consider is the fact that different mixes of power generation come with different risk and variability characteristics. This could be another interesting feature to incorporate in comparing a TPS to a CAP.
References


5 Figures

Figure 1: Fuel Switching as a form of emissions abatement

Figure 2: Definition of the BAU Steady-State using Marginal Investment Cost curves
Figure 3: Phase 0 - $\sigma = \sigma_0$ such that $\Delta PD = \Delta PK_f + \Delta PC_f = 0$

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Figure 5: Phase 1 - $\sigma < \sigma_0$ under perfectly inelastic demand such that $\Delta PD = \Delta PK_f + \Delta PC_f > 0$.

Figure 6: Maps figure 5 into price and quantity space.
Figure 7: Phase 2 - $\sigma < \sigma_0$ under perfectly elastic demand such that $\Delta PD = \Delta PK_f + \Delta PC_f = 0$

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Figure 9: Initial period response to TPS — $\Delta PC_f, \Delta PK_f, \Delta PD$

Initial Period Response to TPS — change from BAU

Figure 10: Path of Investment — $I_{f,t}^2$
Figure 11: Capacity utilization for H

The black horizontal lines indicate the BAU steady-state investment level for both fuel types as they are the same.
Figure 12: Percentage reduction in emissions from BAU

Figure 13: Steady-state response to TPS — $\Delta PC_f$, $\Delta PK_f$, $\Delta PD$
### Table 1: Key Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{d}_t )</td>
<td>100 MW</td>
<td>constant reference demand for each period (t)</td>
</tr>
<tr>
<td>( MC^X_{f,t} )</td>
<td>30 $/MWh</td>
<td>constant operating costs for each fuel (f)</td>
</tr>
<tr>
<td>( \omega_H )</td>
<td>2000 lb CO2e/MWh</td>
<td>emissions intensity for H</td>
</tr>
<tr>
<td>( \omega_L )</td>
<td>0 lb CO2e/MWh</td>
<td>emissions intensity for L</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.05</td>
<td>depreciation/asset decay rate</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>-0.3</td>
<td>elasticity of demand</td>
</tr>
<tr>
<td>( r )</td>
<td>0.01</td>
<td>rate of time preference used for discounting</td>
</tr>
<tr>
<td>( hiy )</td>
<td>8760</td>
<td>operating hours in each period (t)</td>
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### Table 2: Scenario Definition

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Slopes</th>
<th>Policy Stringency</th>
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<tr>
<td>S1A</td>
<td>High/Steep</td>
<td>Low/Loose</td>
</tr>
<tr>
<td>S1B</td>
<td>High/Steep</td>
<td>High/Strict</td>
</tr>
<tr>
<td>S2A</td>
<td>Low/Flat</td>
<td>Low/Loose</td>
</tr>
<tr>
<td>S2B</td>
<td>Low/Flat</td>
<td>High/Strict</td>
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Table 3: Present Value of change in surplus measures for TPS from BAU ($/(hour*year))

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\Delta CS$</th>
<th>$\Delta PS$</th>
<th>$\Delta CInv$</th>
<th>$\Delta AC$</th>
<th>$\Delta Dam$</th>
<th>$\Delta SS_{TPS-CAP}$</th>
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<td>S1A</td>
<td>-13029</td>
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<td>3653</td>
<td>5123</td>
<td>28129</td>
<td>-170</td>
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<td>65162</td>
<td>-105</td>
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<tr>
<td>S2A</td>
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<td>1390</td>
<td>784</td>
<td>956</td>
<td>27568</td>
<td>-34</td>
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<tr>
<td>S2B</td>
<td>-7236</td>
<td>6511</td>
<td>4132</td>
<td>5054</td>
<td>64297</td>
<td>-19</td>
</tr>
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Table 4: Ratio of periodic steady-state social surplus change ($\frac{\Delta SS_{TPS}}{\Delta SS_{CAP}}$)

<table>
<thead>
<tr>
<th>Discount rate ($r$)</th>
<th>Steeper (1)</th>
<th>Flatter (2)</th>
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<tbody>
<tr>
<td>$0$</td>
<td>0.998864</td>
<td>0.999988</td>
</tr>
<tr>
<td>$0.01$</td>
<td>1.004387</td>
<td>1.000573</td>
</tr>
<tr>
<td>$0.03$</td>
<td>1.018866</td>
<td>1.002116</td>
</tr>
</tbody>
</table>
Appendix A  Numerical Model Equations

The discount factor for the purposes of the numerical model is given by:

\[ df_t = \left( \frac{1}{1 + r} \right)^t \] (A.1)

The equations of the numerical model for the TPS with discounting are shown below:

\[ \sum_f X_{ft} \geq Q_t \perp PD_t \geq 0 \] (A.2)

\[ K_{ft} \geq X_{ft} \perp PK_{f,t} \geq 0 \] (A.3)

\[ I_{ft} + K_{f,t-1}(1 - \delta) \geq K_{ft} \perp PI_{f,t} \geq 0 \] (A.4)

\[ \sigma_t \geq \frac{\sum_f \omega_f X_{ft}}{\sum_f X_{ft}} \perp PTPS_t \geq 0 \] (A.5)

\[ df_t \cdot \bar{p}q_t \cdot \left( \frac{Q_t}{d_t} \right)^{1/\epsilon} - PD_t \geq 0 \perp Q_t \geq 0 \] (A.6)

\[ df_t MC^{X}_{ft}(X_{ft}) + PK_{ft} + PTPS_t(\omega_f - \sigma_t)hiy - PD_t \geq 0 \perp X^{TPS}_{ft} \geq 0 \] (A.7)

\[ -PK_{ft} - PI_{f,t+1}(1 - \delta) + PI_{ft} \geq 0 \perp K_{ft} \geq 0 \] (A.8)

\[ df_t MC^{I}_{ft}(I_{ft}) - PI_{ft} \geq 0 \perp I_{ft} \geq 0 \] (A.9)

Where marginal investment costs are linear such that:

\[ MC^{I}_{ft} = [\alpha + \beta_f \cdot I_{f,t}] \] (A.10)

And benchmark output prices a related by:

\[ \bar{p}q_t = \frac{pd_t}{df_t} \] (A.11)
The equations for the model with the CAP are the same except I replace equations A.5 and A.7 with:

\[
\Omega_t \geq \sum_f \omega_f X_{ft} \quad \perp CAP_t \geq 0 \quad (A.12)
\]
\[
df_t MC_{ft}^X (X_{ft}) + PK_t + CAP_t \omega_f h_f y - PD_t \geq 0 \quad \perp X_{ft}^{CAP} \geq 0 \quad (A.13)
\]

**Appendix B  Welfare Calculations**

Changes in periodic discounted consumer surplus are given by:

\[
\Delta CS_t = -d_t PD_t^{-\epsilon} \left( \bar{\omega}_t PD_t - \bar{\omega}_t PD_t \right) / (\epsilon - 1) \quad (B.1)
\]

Changes in periodic discounted producer surplus are given by:

\[
PS_t = PD_t \sum_f X_{ft} - df_t \sum_f MC_{ft}^X X_{ft} \quad (B.2)
\]

\[
\Delta PS_t^{policy} = PS_t^{policy} - PS_t^{BAU} \quad (B.3)
\]

where marginal costs are constant. Revenues from the CAP are refunded to producers, leaving periodic changes in total policy cost equal to 0 when summed across fuel types. Policy costs are also 0 when summed across fuel types for the TPS.

Investment costs are obtained by integrating the marginal investment cost function from 0 to \(I_{ft}\). The function is given below:

\[
CInv_t = df_t \sum_f \left[ \alpha_f I_{ft} + \frac{1}{2} \beta_f I_{ft}^2 \right] \quad (B.4)
\]

Change is analogous to that of \(\Delta PS\) in that:

\[
\Delta CInv_t^{policy} = CInv_t^{policy} - CInv_t^{BAU} \quad (B.5)
\]
Change in periodic discounted avoided damages is given by:

\[ Dam_t = 20 \ast mtCO^2_t \ast df_t \]  \hspace{1cm} (B.6)

\[ \Delta Dam_t = Dam_t^{Bau} - Dam_t^{policy} \] \hspace{1cm} (B.7)

Each of the equations listed thus far are indexed by time, hence, they are periodic discounted flows of the respective changes from the benchmark. In order to calculate the total present value of each figure, I sum from the present to the steady state, and then take the value for the final period and discount it perpetually as follows:

\[ \Delta CS = \sum_{t=0}^{T} \Delta CS_t + \frac{\Delta CS_T}{r} \] \hspace{1cm} (B.8)

where \( T \) is a period within the steady-state. This same formula applies for any of the surplus components \( \Delta PS, \Delta CInv, \Delta AC, \Delta Dam, \) or \( \Delta SS \).